

An Evaluation of the Partisan Fairness of the Pennsylvania Legislative Reapportionment Commission's Proposed State House Districting Plan

Christopher Warshaw*

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*Associate Professor, Department of Political Science, George Washington University. warshaw@gwu.edu. Note that the analyses and views in this report are my own, and do not represent the views of George Washington University.

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1 Introduction

My name is Christopher Warshaw. I am an Associate Professor of Political Science at George Washington University. Previously, I was an Associate Professor at the Massachusetts Institute of Technology from July 2016 - July 2017, and an Assistant Professor at MIT from July 2012 - July 2016.

I have been asked by counsel representing the House Democratic Caucus to analyze relevant data and provide my expert opinions to the Legislative Reapportionment Commission (LRC) about its proposed State House districting plan. I look forward to making a presentation to the LRC on January 14th.

2 Qualifications and Publications

My Ph.D. is in Political Science, from Stanford University, where my graduate training included courses in political science and statistics. I also have a J.D. from Stanford Law School. My academic research focuses on public opinion, representation, elections, and polarization in American Politics. I have written over 20 peer reviewed papers on these topics. Moreover, I have written multiple papers that focus on elections and two articles that focus specifically on partisan gerrymandering. I also have a forthcoming book that includes an extensive analysis on the causes and consequences of partisan gerrymandering in state governments.

My curriculum vitae is attached to this report. All publications that I have authored and published appear in my curriculum vitae. My work is published or forthcoming in peer-reviewed journals such as: the *American Political Science Review*, the *American Journal of Political Science*, the *Journal of Politics*, *Political Analysis*, *Political Science Research and Methods*, the *British Journal of Political Science*, the *Annual Review of Political Science*, *Political Behavior*, *Legislative Studies Quarterly*, *Science Advances*, the *Election Law Journal*, *Nature Energy*, *Public Choice*, and edited volumes from Cambridge University Press and Oxford University Press. My book entitled *Dynamic Democracy in the American States* is forthcoming from the University of Chicago Press. My non-academic writing has been published in the *New York Times* and the *Washington Post*. My work has also been discussed in the *Economist* and many other prominent media outlets.

My opinions in this case are based on the knowledge I have amassed over my education, training and experience, including a detailed review of the relevant academic literature. They also follow from statistical analysis of the following data:

- In order to calculate partisan bias in state house elections on the proposed plan in Pennsylvania, I examined:
 - GIS Files with the 2014-2020 Pennsylvania State House plan and the proposed 2022-30 plan): I obtained both plans from the Legislative Reapportionment Commission’s website.
 - Precinct-level data on recent statewide Pennsylvania elections: I use precinct-level data on Pennsylvania’s statewide elections between 2016-20 from the Voting and Election Science Team (University of Florida, Wichita State University). I obtained these data from the Harvard Dataverse.¹ I obtained precinct-level data on elections from 2012-14 from the MGGG Redistricting Lab.² Finally, I obtained data on state legislative election results from the House Democratic Caucus since they were not available from public sources.
 - A large canonical data set on candidacies and results in state legislative elections: I obtained results from 1972-2020 collected by Carl Klarner and a large team of collaborators. The results from 1972-2012 are based on data maintained by the Inter-university Consortium for Political and Social Research (ICPSR) (Klarner et al. 2013). The data from 2013-2020 were collected by Klarner.
 - Data on presidential election returns in state legislative districts: For elections between 1972 and 1991, I used data on county-level presidential election returns from 1972-1988 collected by the Inter-university Consortium for Political and Social Research (ICPSR 2006) and mapped these returns to state legislative districts. For elections between 1992 and 2001, I used data on presidential election returns in the 2000 election collected by McDonald (2014) and Wright et al. (2009). For elections between 2002 and 2011, I used data on the 2004 and 2008 presidential elections collected by Rogers (2017). For elections between 2012 and 2020, I used data on presidential election returns from the DailyKos website and PlanScore.org.
 - The Plan Score website: PlanScore is a project of the nonpartisan Campaign Legal Center (CLC) that enables people to score proposed maps for their partisan, demographic, racial, and geometric features. I am on the social science advisory team for PlanScore.
- In order to compare the maps in Pennsylvania to congressional elections, I examined:

1. See <https://dataverse.harvard.edu/dataverse/electionscience>.

2. See <https://github.com/mggg-states/PA-shapefiles>.

- A large data set on candidacies and results in Congressional elections: I obtained results from 1972-2018 collected by the Constituency-Level Elections Archive (CLEA) (Kollman et al. 2017). The results from 1972-1990 are based on data collected and maintained by the Inter-university Consortium for Political and Social Research (ICPSR) and adjusted by CLEA. The data from 1992-2018 are based on data collected by CLEA from the Office of the Clerk at the House of the Representatives. I supplemented this dataset with recent election results collected by the MIT Election and Data Science Lab (MIT Election and Data Science Lab 2017) and Dave Leip’s Atlas of U.S. Presidential Elections.
- Data on presidential election returns and incumbency status in Congressional elections. I used data on elections in congressional districts from 1972-2020 collected by Professor Gary Jacobson (University of California, San Diego). This dataset has been used in many Political Science studies and has canonical status in the political science profession (Jacobson 2015).

I have previously provided expert reports in six redistricting-related cases:

- Between 2017 and 2019, I provided reports for *League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania*, No. 159 MM 2017, *League of Women Voters of Michigan v. Johnson*, 17-14148 (E.D. Mich), and *APRI et al. v. Smith et al.*, No. 18-cv-357 (S.D. Ohio). My testimony was found to be credible in each of these cases and was extensively cited by the judges in their decisions.
- In the current redistricting cycle, I have provided reports in *League of Women Voters v. Ohio Redistricting Commission*, No. 2021-1193, *League of Women Voters vs. Kent County Apportionment Commission*, and *League of Women Voters of Ohio v. Ohio Redistricting Commission*, No. 2021-1449.

In addition, I have provided expert testimony and reports in several cases related to the U.S. Census: *State of New York et al. v. United States Department of Commerce*, 18-cv-2921 (S.D.N.Y.), *New York v. Trump*; *Common Cause v. Trump*, 20-cv-2023 (D.D.C.), and *La Union Del Pueblo Entero (LUPE) v. Trump*, 19-2710 (D. Md.).

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3 Summary

The relationship between the distribution of partisan support in the electorate and the partisan composition of the government—what Powell (2004) calls “vote–seat representation”—is a critical link in the longer representational chain between citizens’ preferences and governments’ policies. If the relationship between votes and seats systematically advantages one party over another, then some citizens will enjoy more influence—more “voice”—over elections and political outcomes than others (Caughey, Tausanovitch, and Warshaw 2017).

I use three complementary methodologies to project future election results in order to evaluate the partisan fairness of Pennsylvania’s proposed State House plan. First, I use a composite of previous statewide election results between 2014-2020 to analyze the new map.³ Second, I analyze the results of the 2020 State House election on the newly proposed map. Third, I complement this approach using the open source PlanScore.org website, which is a project of the Campaign Legal Center.⁴ PlanScore uses a statistical model to estimate district-level vote shares for a new map based on the relationship between presidential election results and legislative results between 2014-2020.⁵ Based on these three approaches, I characterize the bias in Pennsylvania’s plans based on a large set of established metrics of partisan fairness and place the bias in Pennsylvania’s plans into historical perspective. I also analyze whether the proposed plan is responsive to shifts in voters’ preferences.

All of these analyses indicate that the proposed map is fair with just a small pro-Republican bias. Indeed, one important feature of the proposed plan is that it enables the party that wins the majority of the votes to nearly always win the majority of the seats. In the actual 2020 State House election, Republicans received 50.5% of the two-party vote and Republicans would win 50.2% of the seats in the proposed plan.⁶ In the 2020 presidential election, Democrat Joe Biden received about 50.6% of the two-party vote and he would have won 102 out of the 203 (50.2%) of the State House districts.⁷ Based on the statewide elections in Pennsylvania between 2014-2020, the Democrats’ statewide two-party vote share averaged about 54% of the vote and they would win nearly exactly

3. These include the following elections: 2016 Presidential, 2020 Presidential, 2014 Governor, 2018 Governor, 2016 Attorney General, 2020 Attorney General, 2016 Senate, 2018 Senate, 2016 Treasurer, 2020 Treasurer, 2016 Auditor, and 2020 Auditor election.

4. I am on the social science advisory board of Plan Score, but do not have any role in PlanScore’s evaluation of individual maps.

5. See <https://planscore.campaignlegal.org/models/data/2021D/> for more details.

6. I impute uncontested State House elections using the presidential election results.

7. Following standard convention, throughout my analysis I focus on two-party vote shares.

the same proportion of the seats on the proposed plan (54.5%).⁸ Historically, there is a winner’s bonus where the party that wins 54% of the votes typically receives about 58% of the seats. So recent statewide elections indicate a modest pro-Republican bias in the plan using a wide variety of Political Science metrics for partisan fairness.

I also reach the conclusion that the plan is relatively neutral, with a small pro-Republican bias, using the predictive model on the PlanScore website. PlanScore projects that Republicans would get about 50.3% of the statewide vote, but Republicans are expected to win 53% of the seats in Pennsylvania’s proposed State House plan (and Democrats would win 47% of the seats).⁹ Across 1000 simulations, PlanScore indicates that the proposed plan favors Republican candidates in 95% of scenarios. Based on generally accepted Political Science metrics for partisan fairness, PlanScore indicates that Pennsylvania’s proposed plan would have a modest level of pro-Republican bias.

The remainder of the report proceeds as follows. First, I discuss partisan gerrymandering and how social scientists measure partisan bias in a districting plan. I also discuss how to conceptualize the responsiveness of a districting plan to shifts in voters’ preferences. Next, I examine the partisan fairness of the proposed State House plan, and compare it to the fairness of other plans around the country over the past 50 years. Then, I examine the responsiveness of the proposed plan to shifts in voters’ preferences and the number of competitive districts in the proposed plans. Finally, I briefly conclude.

4 Background on Partisan Fairness

This section provides background about how social scientists conceptualize partisan fairness in a districting plan. Partisan advantage in a districting plan may arise either intentionally, due to a deliberate effort to benefit the line-drawing party and handicap the opposing party via gerrymandering (Kang 2017; Levitt 2017), or unintentionally as a result of factors such as political geography, candidate appeal, and electoral swings (Chen and Rodden 2013; Goedert 2014; Seabrook 2017). Whether districting bias is purposeful or accidental, it means that one party’s voters are more “cracked” and “packed” than the other side’s supporters. In cracked districts, voters’ preferred candidates lose by relatively narrow margins; in packed districts, their candidates of choice win by enormous margins

8. I weight the composite scores to give each election cycle equal weight in the index. The seat-level projections are based on the 12 statewide elections where I have precinct-level data. If instead I use the approach that Professor Michael Barber references in his report and simply average across contests, Democrats win 52% of the votes and 52% of the seats on the proposed plan.

9. This is a probabilistic estimate based on 1000 simulations of possible elections using a model of the elections between 2014-2020.

(Stephanopoulos and McGhee 2015). Thanks to disproportionate cracking and packing, the disfavored party is less able than the favored party to convert its statewide support among voters into legislative representation. This gives the favored party the ability to shift policies in its direction (Caughey, Tausanovitch, and Warshaw 2017) and build a durable advantage in downstream elections (Stephanopoulos and Warshaw 2020). It can even lead to undemocratic outcomes where the advantaged party wins the majority of the seats and controls the government while only winning a minority of the votes.

There are a number of approaches that have been proposed to measure partisan advantage in a districting plan. These approaches focus on asymmetries in the efficiency of the vote–seat relationships of the two parties. In recent years, at least 10 different approaches have been proposed (McGhee 2017). While no measure is perfect, much of the recent literature has focused on four related approaches that I describe below.

4.1 Symmetry in the Vote-Seat Curve Across Parties

Basic fairness suggests that in a two-party system each party should receive the same share of seats for identical shares of votes. The *symmetry* idea is easiest to understand at an aggregate vote share of 0.5—a party that receives half the vote ought to receive half the seats—but a similar logic can apply across the “seats-votes curve” that traces out how seat shares change as vote shares rise and fall. For example, if a party receives a vote share of 0.57 and a seat share of 0.64, the opposing party should also expect to receive a seat share of 0.64 if it were to receive a vote share of 0.57. An unbiased system means that for V share of the votes a party should receive S share of the seats, and this should be true for all parties and vote percentages (Niemi and Deegan 1978; Gelman and King 1994; McGhee 2014; Katz, King, and Rosenblatt 2020).

Gelman and King (1994, 536) propose two ways to measure partisan bias in the symmetry of the vote-seat curve. First, it can be measured using counter-factual election results in a range of statewide vote shares between .45 and .55. Across this range of vote shares, each party should receive the same number of seats. Symmetry captures any departures from the standard that each party should receive the same seat share across this range of plausible vote shares. For example, if partisan bias is -0.05, this means that the Democrats receive 5% fewer seats in the legislature than they should under the symmetry standard (and the Republicans receive 5% more seats than they should). Second, symmetry can be measured based on the seat share that each party receives when they split the statewide vote 50-50. In an unbiased system, each party should receive 50% of the seats in a tied statewide election. Here, the partisan bias statistic is the “expected

proportion of the seats over 0.5 that the Democrats receive when they receive exactly half the average district vote.”

To illustrate the symmetry metric, Figure 1 shows what each party’s share of the seats would have been across a range of statewide vote shares from 45%-55%. The left-hand panel shows the gerrymandered 2016 US House election. On this plan, Democrats received 22% of the seats when they received 45% of the statewide vote, 28% of the seats when they won half the vote, and just 33% of the seats when they received 55% of the statewide vote. In contrast, Republicans received 66% of the seats when they received 45% of the vote, 72% of the seats when they won half the vote, and 78% of the seats when they received 55% of the vote. This indicates a historically extreme pro-Republican symmetry bias of about -20%.

The right-hand panel of Figure 1 shows the proposed State House plan (using re-aggregated votes in the 2020 State House Elections). On this plan, Democrats would receive about 45% of the seats when they receive 45% of the votes, 49.8% of the seats when they win half the vote, and 54% of the seats when they receive 55% of the votes. Republicans would receive about 46% of the seats when they receive 45% of the votes, 50.2% of the seats when they win half the vote, and 55% of the seats when they receive 55% of the votes. This indicates an almost perfectly fair plan using the symmetry metric with virtually no bias.

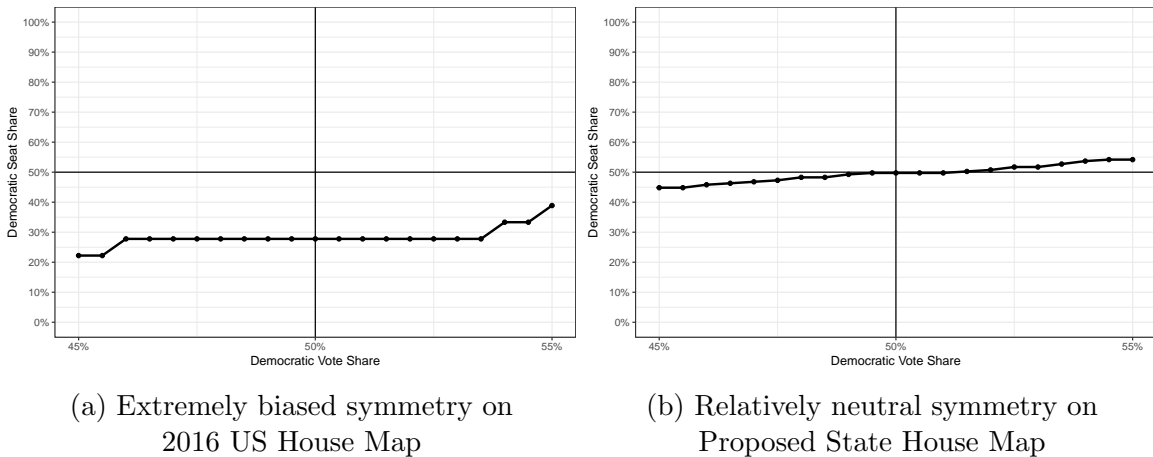


Figure 1: Plot illustrating an extremely asymmetrical map based on 2016 US House election and the more symmetrical proposed plan using re-aggregated votes in 2020 Pennsylvania State House Elections

A weakness of the symmetry approach is that it requires the analyst to calculate counterfactual elections. This approach has both conceptual and empirical limitations. At a conceptual level, it is not clear that it aligns perfectly with the usual definition of a

gerrymander. Indeed, “when observers assert that a district plan is a gerrymander, they usually mean that it systematically benefits a party (and harms its opponent) in actual elections. They do not mean that a plan would advantage a party in the hypothetical event of a tied election, or if the parties’ vote shares flipped” (Stephanopoulos and McGhee 2015, 857). At an empirical level, in order to generate symmetry metrics, we need to simulate counter-factual elections by shifting the actual vote share in each district a uniform amount (McGhee 2014).¹⁰ In general, this uniform swing assumption seems reasonable based on past election results (though is probably less reasonable in less competitive states). Moreover, it has been widely used in past studies of redistricting. But there is no way to conclusively validate the uniform swing assumption for any particular election.

An important strength, however, of the symmetry approach is that it is based on the shape of the seats-votes curve and not any particular point on it. As a result, it is relatively immune to shifts in party performance (McGhee 2014). For instance, the bias toward Republicans in Pennsylvania’s State House elections was very similar in 2014–2020. Moreover, the symmetry approach has been very widely used in previous studies of gerrymandering and redistricting (Gelman and King 1994; McGhee 2014). Overall, the symmetry approach is useful for assessing partisan advantage in the districting process.

4.2 Mean-median Gap

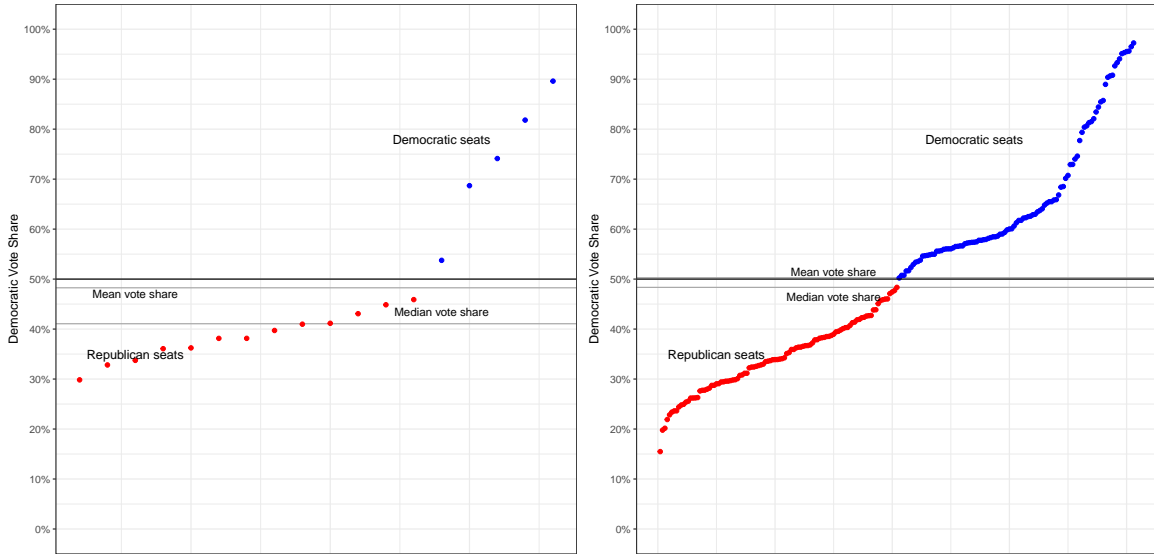
Another metric that some scholars have proposed to measure partisan bias in a districting plan is the *mean-median gap*: the difference between a party’s vote share in the median district and their average vote share across all districts. If the party wins more votes in the median district than in the average district, they have an advantage in the translation of votes to seats (Krasno et al. 2018; Best et al. 2017; Wang 2016). In statistics, comparing a dataset’s mean and median is a common statistical analysis used to assess skews in the data and detect asymmetries (Brennan Center 2017).

The mean-median difference is very easy to apply (Wang 2016). It is possible, however, for packing and cracking to occur without any change in the mean-median difference (Buzas and Warrington 2021). That is, a party could gain seats in the legislature without the mean-median gap changing (McGhee 2017).¹¹ It is also sensitive to the outcome in

10. In principle, the uniform swing election could be relaxed, and swings could be estimated on a district-by-district basis. But this is rarely done in practice since it would require a much more complicated statistical model, and probably would not improve estimates of symmetry very much.

11. As McGhee (2017), notes, “If the median equals the win/loss threshold—i.e., a vote share of 0.5—then when a seat changes hands, the median will also change and the median-mean difference will reflect that change. But if the median is anything other than 0.5, seats can change hands without any change in the median and so without any change in the median-mean difference.” See also Buzas and Warrington

the median district (Warrington 2018b). In addition, the mean-median difference lacks a straightforward interpretation in terms of the number of seats that a party gains through gerrymandering.



(a) Extremely biased mean-median difference on 2016 US House Map (b) Relatively neutral mean-median diff. on Proposed State House Map

Figure 2: Plot illustrating an extremely biased mean-median difference based on 2016 US House election and a more neutral mean-median difference on the proposed plan using re-aggregated votes in 2020 Pennsylvania State House Elections

Figure 2 illustrates the mean-median difference. The left-hand panel shows the 2016 US House elections. In this election, the mean-median difference was about -7.5%. This means that Republicans did about 7.5% better in the median seat than statewide, which gave them a large advantage in the translation of votes to seats. The right-hand panel shows the proposed State House plan (using re-aggregated votes in the 2020 State House Elections). Across all districts, Democrats won an average of 50.3% of the vote. But they only won 48.3% in the median district. So the mean-median difference here was -1.9%. It still favors Republicans, but much less than on the heavily gerrymandered 2012-16 US House plan.

(2021) who make a similar point using simulated packing and cracking.

4.3 Efficiency Gap

Both cracked and packed districts “waste” more votes of the disadvantaged party than of the advantaged one (McGhee 2014; Stephanopoulos and McGhee 2015).¹² This suggests that gerrymandering can be measured based on asymmetries in the number of wasted votes for each party. The *efficiency gap* (EG) focuses squarely on the number of each party’s wasted votes in each election. It is defined as “the difference between the parties’ respective wasted votes, divided by the total number of votes cast in the election” (Stephanopoulos and McGhee 2015, 831; see also McGhee 2014, 2017). All of the losing party’s votes are wasted if they lose the election. When a party wins an election, the wasted votes are those above the 50%+1 needed to win.

If we adopt the convention that positive values of the efficiency gap imply a Democratic advantage in the districting process and negative ones imply a Republican advantage, the efficiency gap can be written mathematically as:

$$EG = \frac{W_R}{n} - \frac{W_D}{n} \quad (1)$$

where W_R are wasted votes for Republicans, W_D are wasted votes for Democrats, and n is the total number of votes in each state.

Table 1 provides a simple example about how to calculate the efficiency gap with three districts where the same number of people vote in each district. In this example, Democrats win a majority of the statewide vote, but they only win 1/3 seats. In the first district, they win the district with 75/100 votes. This means that they only wasted the 24 votes that were unnecessary to win a majority of the vote in this district. But they lose the other two districts and thus waste all 40 of their votes in those districts. In all, they waste 104 votes. Republicans, on the other hand, waste all 25 of their votes in the first district. But they only waste the 9 votes unnecessary to win a majority in the two districts they win. In all, they only waste 43 votes. This implies a pro-Republican efficiency gap of $\frac{43}{300} - \frac{104}{300} = -20\%$.

In order to account for unequal population or turnout across districts, the efficiency gap formula in equation 1 can be rewritten as:

$$EG = S_D^{margin} - 2 * V_D^{margin} \quad (2)$$

where S_D^{margin} is the Democratic Party’s seat margin (the seat share minus 0.5) and V_D^{margin}

12. The authors of the efficiency gap use the term “waste” or “wasted” to describe votes for the losing party and votes for the winning party in excess of what is needed to win an election. Since the term is used by the efficiency gap authors, I use it here when discussing the efficiency gap.

Table 1: Illustrative Example of Efficiency Gap

District	Democratic Votes	Republican Votes
1	75	25
2	40	60
3	40	60
Total	155 (52%)	145 (48%)
Wasted	104	43

is is the Democratic Party’s vote margin. V_D^{margin} is calculated by aggregating the raw votes for Democratic candidates across all districts, dividing by the total raw vote cast across all districts, and subtracting 0.5 (McGhee 2017, 11-12). In the example above, this equation also provides an efficiency gap of -20% in favor of Republicans. But it could lead to a slightly different estimate of the efficiency gap if districts are malapportioned or there is unequal turnout across districts.¹³

In the case of Pennsylvania’s proposed State House map, equation 2 implies there would have been a pro-Democratic efficiency gap of 0.7% using the votes from the 2020 election re-aggregated onto the proposed plan. This is very close to the middle of the distribution of previous Efficiency Gaps in state legislative elections.

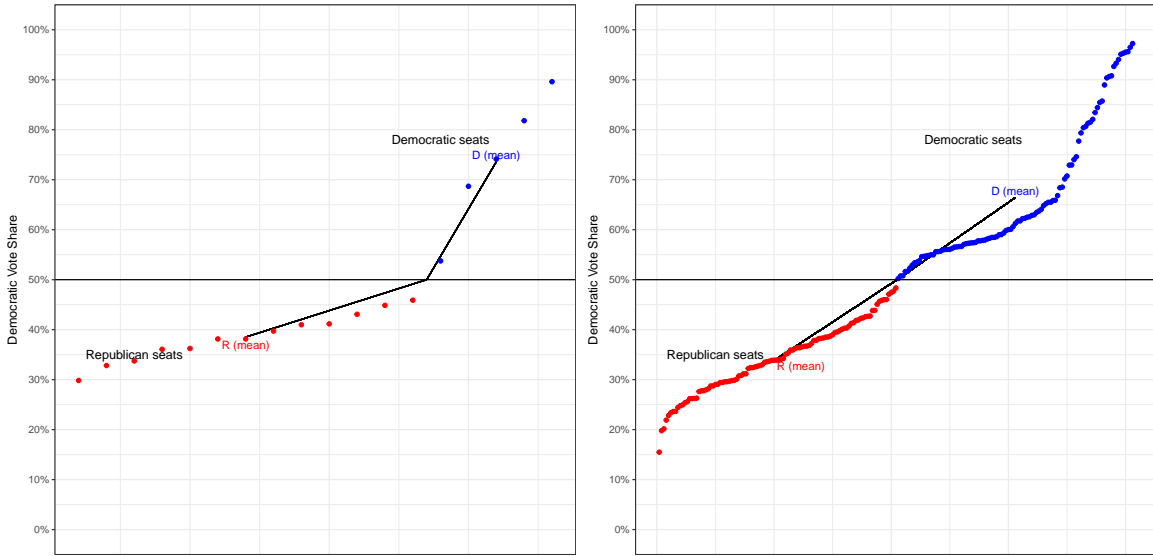
The efficiency gap mathematically captures the packing and cracking that are at the heart of partisan gerrymanders (Buzas and Warrington 2021). It measures the extra seats one party wins over and above what would be expected if neither party were advantaged in the translation of votes to seats (i.e., if they had the same number of wasted votes). A key advantage of the efficiency gap over other measures of partisan bias is that it can be calculated directly from observed election returns even when the parties’ statewide vote shares are not equal.

The symmetry metric is closely related to the efficiency gap. In the special case where each party receives half of the statewide vote, the symmetry and the efficiency gap metrics are mathematically identical (Stephanopoulos and McGhee 2015, 856). More generally, the symmetry and efficiency gap yield very similar substantive results when each party’s statewide vote share is close to 50% (as is the case in Pennsylvania). When elections are uncompetitive, however, and one party wins a large percentage of the statewide vote, the efficiency gap and these symmetry metrics are less correlated with one another (857).

13. In general, the two formulations of the efficiency gap formula yield very similar results. Because Democrats tend to win lower-turnout districts, however, the turnout adjusted version of the efficiency gap in equation 2 tends to produce results that suggest about a 2% smaller disadvantage for Democrats than the version in Equation 1 (see McGhee 2018).

4.4 Declination

Another measure of asymmetries in redistricting plans is called *declination* (Warrington 2018b, 2018a). The declination metric treats asymmetry in the vote distribution as indicative of partisan bias in a districting plan (Warrington 2018a). If all the districts in a plan are lined up from the least Democratic to the most Democratic, the mid-point of the line formed by one party’s seats should be about as far from the 50 percent threshold for victory on average as the other party’s (McGhee 2018).



(a) Extremely biased declination on 2016 US House Map

(b) Relatively neutral declination on Proposed State House Map

Figure 3: Plot illustrating an extremely biased declination based on 2016 US House election and a fair declination on proposed plan using re-aggregated votes in 2020 Pennsylvania State House Elections

Declination suggests that when there is no gerrymandering, the angles of the lines (θ_D and θ_R) between the mean across all districts and the point on the 50% line between the mass of points representing each party will be roughly equal. When they deviate from each other, the smaller angle (θ_R in the case of Pennsylvania) will generally identify the favored party. To capture this idea, declination takes the difference between those two angles (θ_D and θ_R) and divides by $\pi/2$ to convert the result from radians to fractions of 90 degrees.¹⁴ This produces a number between -1 and 1. As calculated here, positive values favor Democrats and negative values favor Republicans. Warrington (2018b) suggests a further adjustment to account for differences in the number of seats across legislative

14. This equation is: $\delta = 2 * (\theta_R - \theta_D) / \pi$.

chambers. I use this adjusted declination estimate in the analysis that follows.¹⁵

Figure 3 illustrates the declination metric. The left-hand panel shows the 2016 US House elections, which was an historically extreme pro-Republican gerrymander. Here, it is easy to see that the angle of the line between the x-axis and the average Republican seat is much less steep than the line between the x-axis and the average Democratic seat. The right-hand panel shows the proposed State House plan (using re-aggregated votes in the 2020 State House Elections). In this plot, the slope of the lines to the Democratic and Republican seats are nearly equal. Thus, the declination metric indicates that the plan has a nearly perfectly neutral declination of -.04.

4.5 Comparison of Partisan Bias Measures

All of the measures of partisan advantage discussed in the previous sections are closely related both theoretically and empirically (McGhee 2017; Stephanopoulos and McGhee 2018). Broadly speaking, all of the metrics consider how votes between the two parties are distributed across districts (Warrington 2018a). For example, the efficiency gap is mathematically equivalent to partisan bias in tied statewide elections (Stephanopoulos and McGhee 2018). Also, the median-mean difference is similar to the symmetry metric, since any perfectly symmetric seats-votes curve will also have the same mean and median (McGhee 2017).

Second, each of the concepts are closely related empirically, particularly in states with competitive elections. Figure 4 shows the correlation between each measure. The various measures have high correlations with one another.¹⁶ Moreover, most of the variation in the metrics can be summarized on a single latent dimension (Stephanopoulos and McGhee 2018; Stephanopoulos and Warshaw 2020). So, overall, while there may be occasional cases where the metrics disagree about the amount of bias in a particular plan, the various metrics usually yield similar results for the degree of partisan bias in a districting plan (Nagle 2015). Where none of the metrics is an outlier and they all point in the same direction, we can draw a particularly robust conclusion

15. This adjustment uses this equation: $\hat{\delta} = \delta * \ln(\text{seats}) / 2$

16. While each measure is highly correlated with one another, the efficiency gap and declination measures are particularly closely related and the symmetry and mean-median measures are very closely related. This could be because the efficiency gap and the declination consider the seats actually won by each party, while the symmetry metric and the mean-median difference do not (Stephanopoulos and McGhee 2018, 1557). In addition, the efficiency gap and the declination appear to best capture the packing and cracking that characterize partisan gerrymandering (Buzas and Warrington 2021).

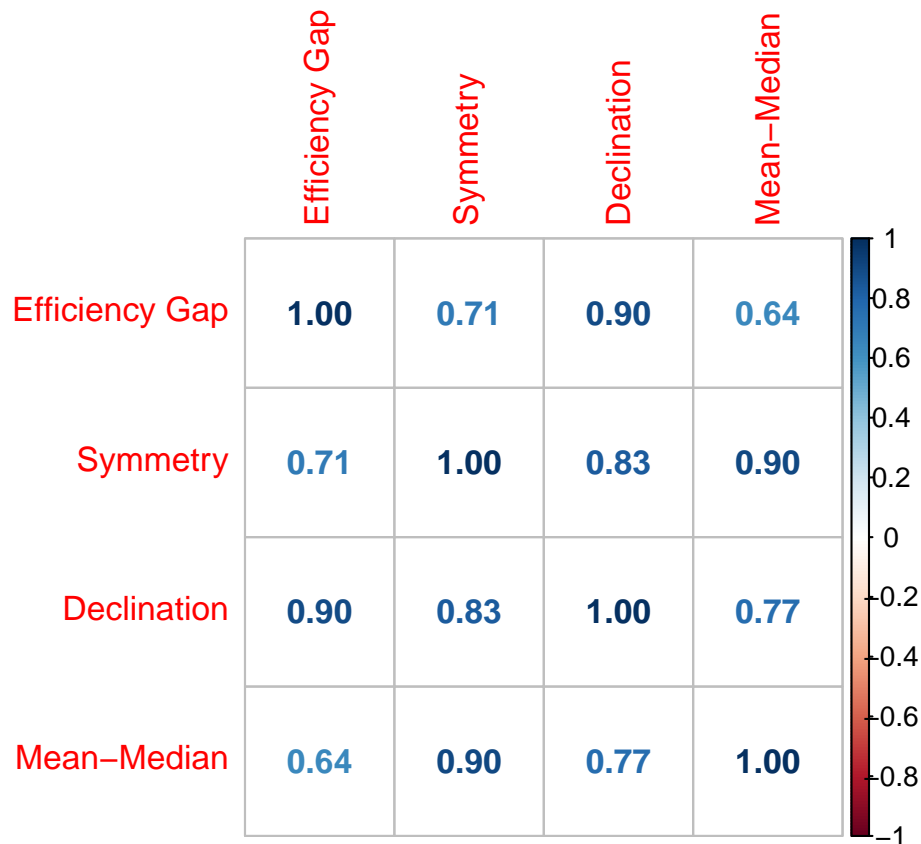


Figure 4: Correlation between measures of partisan bias in competitive states.

4.6 Responsiveness and Competitive Elections

Another benchmark for a districting plan is the percentage of districts likely to have competitive elections under that plan and the responsiveness of the plan to changes in voters' preferences (Cox and Katz 1999). An unresponsive map ensures that the bias in a districting plan toward the advantaged party is insulated against changes in voters' preferences, and thus is durable across multiple election cycles.

To illustrate the concept of responsiveness, Figure 5 shows the vote-seat curve in Pennsylvania for the 2016 US House plan and the proposed State House plan. Similarly to the figure illustrating the symmetry metric, these plots are generated by applying uniform swings to the actual election results.¹⁷ Specifically, I apply a uniform swing in the actual election results until I achieve an average Democratic vote share of 40%. Then

¹⁷. The layout of this chart is adapted from charts in Royden, Li, and Rudensky (2018).

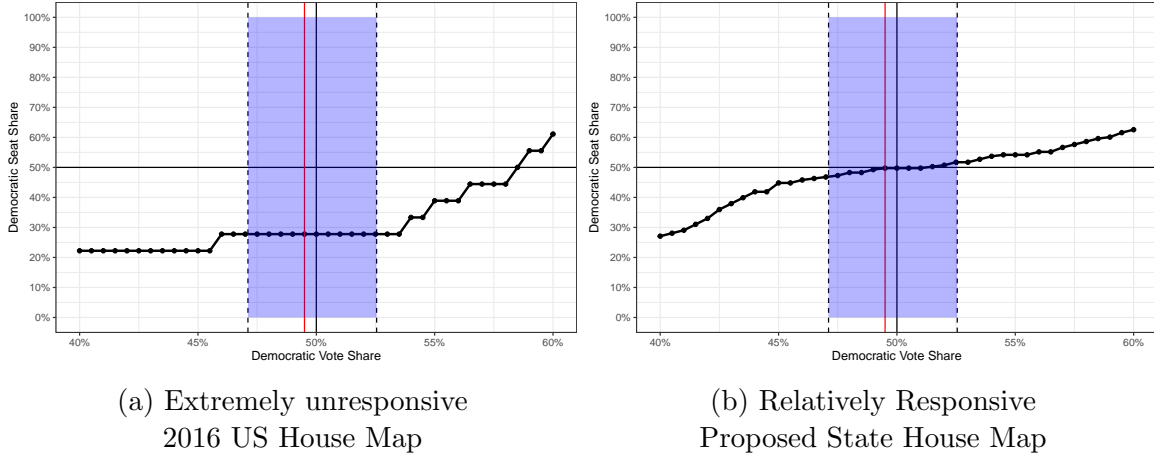


Figure 5: Vote-seat curve in Pennsylvania using uniform swings in 2020 election results re-aggregated using proposed plan. The shaded area shows the range between the minimum and maximum Democratic statewide vote share in State House elections from 2014-2020. The red line shows the actual Democratic statewide vote share in the 2020 State House elections.

I steadily increase the average Democratic vote share until it reaches 60%. The shaded area shows the range between the minimum and maximum Democratic statewide vote share in State House elections from 2014-2020. The red line shows the actual Democratic statewide vote share in the 2020 State House elections.

The left panel of Figure 5 indicates that Republicans win two thirds or more of the US House seats across all of the range of actual election swings over the past decade. In contrast, the proposed State House plan is relatively responsive to changes in statewide preferences. The Democratic seat share increases by 5 percentage points across the range of actual election results and about 10 percentage points as their statewide vote share goes from 45 to 55 percentage points.

An important factor that affects the overall responsiveness of a plan is the number of competitive districts in a plan. First, this affects the responsiveness of a map as the two parties' statewide vote shares rise and fall. A plan with more competitive elections is likely to be more responsive to changes in voters' preferences than a plan with fewer competitive elections (McGhee 2014). Second, uncompetitive districts tend to protect incumbents from electoral sanctions (Tufte 1973; Gelman and King 1994). This could harm political representation by making legislators less responsive and accountable to their constituents' preferences.

There are a couple of approaches we might use to evaluate whether individual districts on a plan are likely to have competitive elections. We could measure whether a district was competitive in an election based on whether the winning party received less than 55%

of the two-party vote (Fraga and Hersh 2018; Jacobson and Carson 2015, 91).¹⁸ While this definition is sometimes used in the literature, though, it is not clear that a sharp threshold at 55% is the best measure of competitiveness.

Another possible definition of competitiveness might be whether a district is likely to switch parties at least once per decade (Henderson, Hamel, and Goldzimer 2018). This definition is more empirically robust because it is not dependent on any particular electoral threshold for competitiveness. Indeed, in a state with swing voters where the two parties' statewide shares vary substantially over the course of the decade, a district where the winning party normally wins 56% of the vote could be competitive. In another state with few swing voters and very inelastic election results, a district where the winning party normally wins 53% of the vote might not even be competitive.

5 Partisan Fairness of Pennsylvania's proposed State House Map

In this section, I will provide a comprehensive evaluation of the partisan fairness of Pennsylvania's proposed State House districting plan (see Figure 6 for a map of the proposed plan). In order to evaluate the proposed plan, we need to predict future election results on this map. Unfortunately, there is no way to know, with certainty, the results of future elections. Thus, I use three complementary methodologies to predict future State House elections in Pennsylvania and generate the various metrics I discussed earlier.

5.1 Composite of previous statewide elections

First, I use a composite of previous statewide election results between 2014-2020 re-aggregated to the proposed map.¹⁹ For each year, I estimate each party's vote share, seat share, and the average of the partisan bias metrics across races. I then average them together to produce a composite result. This approach implicitly assumes that future voting patterns will look like the average of these recent statewide elections.

When I average across these statewide elections from 2014-2020, Democrats win 54% of the votes and 54% of the seats on the proposed plan (see Table 2).²⁰ Thus, the plan

18. Fraga and Hersh (2018) justify this definition based on the fact that the Cook Political Report's "median 'leaning' race ended up with a vote margin of 10 percentage points (a 55%–45% race)."

19. These include the following elections: 2016 Presidential, 2020 Presidential, 2014 Governor, 2018 Governor, 2016 Attorney General, 2020 Attorney General, 2016 Senate, 2018 Senate, 2016 Treasurer, 2020 Treasurer, 2016 Auditor, and 2020 Auditor election.

20. I weight the composite scores to give each election cycle equal weight in the index. The seat-level

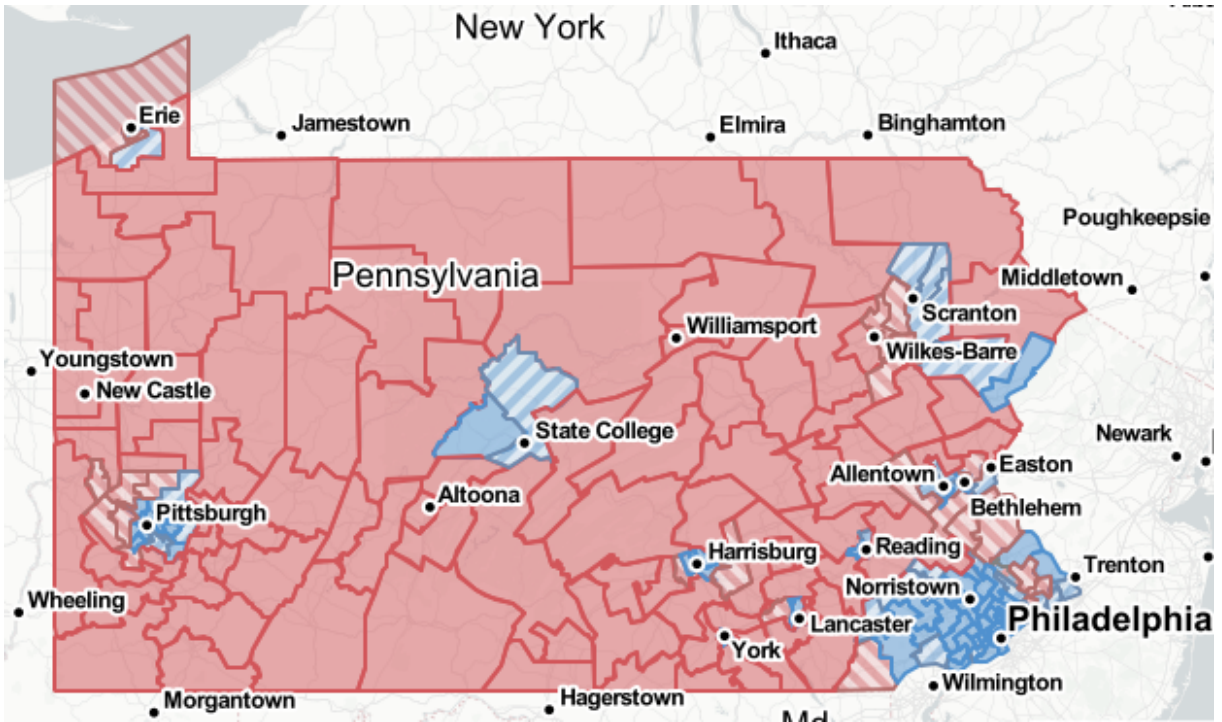


Figure 6: Map of proposed State House Districts from PlanScore.org

Metric	Value	2014-2020 Composite	
		> Biased than this % Elections	> Pro-Rep. than this % Elections
2014-2020 Plan			
Symmetry Bias	-7.7%	77%	85%
Mean-Median	-3.8%	70%	81%
Efficiency Gap	-5.8%	60%	83%
Declination	-.348	66%	82%
Average		68%	83%
Proposed Plan			
Symmetry Bias	-2.5%	29%	61%
Mean-Median	-1.4%	31%	63%
Efficiency Gap	-2.6%	27%	69%
Declination	-.175	38%	65%
Average		31%	65%

Table 2: Composite bias metrics for proposed plan based on statewide elections

satisfies the principal that the party that wins a significant majority of the statewide vote should also win a majority of the seats. However, Democrats did unusually well in these recent statewide elections. In state legislative elections, the two parties typically get closer to 50% of the statewide vote. Thus, another important benchmark is to examine what happens if each party evenly splits the votes. Basic fairness suggests that when the two projections are based on the 12 statewide elections where I have precinct-level data.

parties split the votes they should also split the seats. But the composite election index indicates that when Democrats win 50% of the votes on the proposed plan, they are likely to only win 47.5% of the seats. This leads to a pro-Republican bias on the symmetry metric of 2.5%.

The plan also has a small pro-Republican bias on the other metrics I evaluate. For instance, Republicans do about 1.4% better in the median district than in the mean district and Republicans have a 2.6% advantage in the Efficiency Gap. Overall, the plan has a larger pro-Republican bias in the translation of votes to seats than 65% of previous plans over the past 50 years.

5.2 2020 State House election results

Next, I use the 2020 precinct-level State House results on both the 2014-20 map and re-aggregated to the proposed map to estimate the various metrics. This approach implicitly assumes that future elections will look like the 2020 election.²¹ These endogenous election are likely to be an excellent predictor of future voting patterns in State House elections. But it is important to keep in mind that they could be affected by the individual candidates in each race as well as a host of other factors that wouldn't look exactly the same in future elections.

Metric	Value	More Biased than this % Historical Elections	More Pro-Republican than this % Historical Elections
2014-2020 Plan			
Symmetry Bias	-5.7%	60%	77%
Mean-Median Diff	-4.3%	79%	86%
Efficiency Gap	-4.8%	49%	78%
Declination	-.36	68%	83%
Average		64%	81%
Proposed Plan			
Symmetry Bias	-0.2%	2%	49%
Mean-Median Diff	-1.9%	40%	68%
Efficiency Gap	0.7%	8%	51%
Declination	-.04	9%	50%
Average		15%	55%

Table 3: Partisan bias metrics for State House plan based on 2020 State House election results re-aggregated onto proposed map

21. As is commonly done in the academic literature, I impute uncontested State House elections using the presidential election results. In State House district 7, the Democratic candidate won even though former-President Trump won the majority of the vote. In this district, I adjust the presidential vote so that the Democratic vote share is 51% to ensure that the imputed results yield the correct number of Democratic and Republican seats.

The proposed plan is nearly perfectly unbiased based on the re-aggregated 2020 State House results. Republicans would win 50.5% of the votes and 50.2% of the seats on the proposed plan. Moreover, both parties would receive nearly half the seats when the statewide vote is exactly evenly split. Thus, the symmetry bias is only .2%, which is right in the center of the historical distribution of partisan symmetries. The proposed plan is also nearly perfectly neutral using the other metrics. Only the mean-median difference implies a significant Republican advantage in the translation of votes to seats. When we average across all four metrics, the plan is more extreme than 15% of prior plans, and thus more neutral than 85% of prior plans. When I average across the various metrics, it just has a very small pro-Republican advantage: it is more pro-Republican than 55% of previous plans.

5.3 PlanScore

Third, I evaluate the proposed plan using a predictive model from the PlanScore.org website.²² PlanScore uses a statistical model of the relationship between districts’ latent partisanship and legislative election outcomes. This enables it to estimate district-level vote shares for a new map and the corresponding partisan gerrymandering metrics.²³ It then calculates various partisan bias metrics. Like the earlier approaches, PlanScore indicates that the proposed plan is relatively neutral with a small pro-Republican bias (Table 4).

Metric	Value	Favors Rep’s in this % of Scenarios	More Biased than this % Historical Plans	More Pro-Republican than this % Historical Plans
2014-2020 Plan				
Symmetry	-4.5%	99%	50%	72%
Mean-Median Diff.	-2.0%	99%	42%	68%
Efficiency Gap	-4.6%	99%	53%	81%
Declination	-.27	99%	57%	76%
Average		99%	50%	74%
Proposed Plan				
Symmetry	-2.5%	94%	31%	61%
Mean-Median Diff.	-1.2%	94%	27%	61%
Efficiency Gap	-2.5%	95%	32%	70%
Declination	-.15	95%	37%	64%
Average		95%	31%	64%

Table 4: PlanScore partisan bias metrics for proposed State House plan

22. See <https://planscore.campaignlegal.org/plan.html?20211228T165635.851306606Z> for the proposed plan and <https://planscore.campaignlegal.org/plan.html?20220107T194310.216726037Z> for the 2014-2020 plan.

23. See <https://planscore.campaignlegal.org/models/data/2021D/> for more details.

According to PlanScore, the proposed plan has a small pro-Republican symmetry bias of -2.5%. This means that Republicans would win 52.5% of the seats if the two parties evenly split the votes. The proposed plan favors Republicans in 95% of the scenarios estimated by PlanScore. The other metrics look similar to the symmetry metric. Across all the metrics, the proposed plan is more pro-Republican than 64% of prior plans over the past five decades. Figure 7 graphically shows the bias of the proposed plan compared to previous plans from 1972-2020.²⁴ Overall, the graphs show that the proposed plan is close to the center of the distribution of previous plans over the past 50 years with just a small pro-Republican bias.



Figure 7: Graphs of PlanScore metrics proposed State House plan compared to previous plans from 1972-2020

5.4 Responsiveness of Plan

Another benchmark for a districting plan is the responsiveness of the plan to changes in voters' preferences (Cox and Katz 1999). An unresponsive map ensures that the bias in a districting plan toward the advantaged party is insulated against changes in voters' preferences, and thus is durable across multiple election cycles.

24. Note that the PlanScore graphs are oriented so that pro-Republican scores have a positive value.

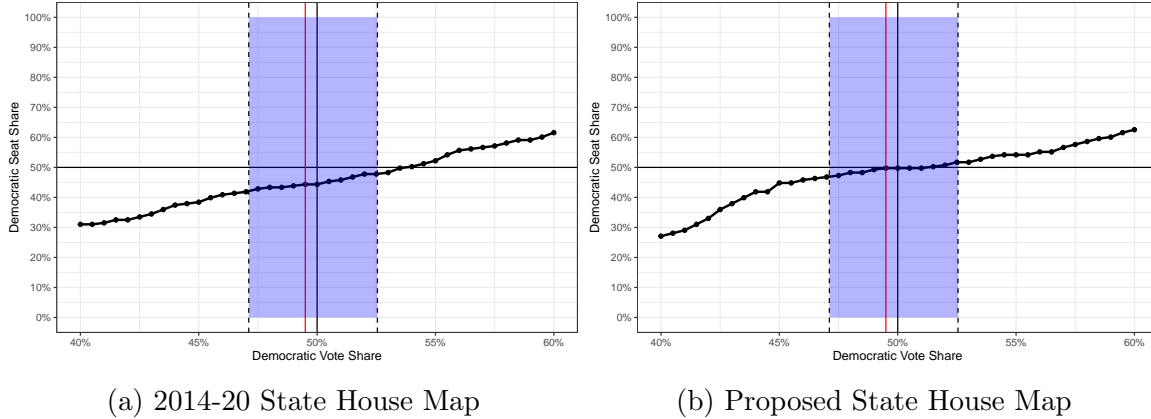


Figure 8: Vote-seat curve in Pennsylvania using uniform swings in 2020 election results on the 2014-20 districts and re-aggregated on the proposed plan. The shaded area shows the range between the minimum and maximum Democratic statewide vote share in State House elections from 2014-2020. The red line shows the actual Democratic statewide vote share in the 2020 State House elections.

Figure 8 compares the responsiveness of the 2014-20 State House plan and the proposed State House plan (using re-aggregated votes in the 2020 State House Elections). It shows the vote-seat curve in Pennsylvania using uniform swings in 2020 election results on the 2014-20 districts and re-aggregated on the proposed plan. The shaded area shows the range between the minimum and maximum Democratic statewide vote share in State House elections from 2014-2020. The red line shows the actual Democratic statewide vote share in the 2020 State House elections.

The graph shows that both the previous plan and the proposed plan are relatively responsive to shifts in voters’ preferences. But the 2014-20 plan had a large pro-Republican bias, which is much smaller in the proposed plan. Indeed, the Republican Party won a majority of the seats across all of the plausible range of stateside vote shares in the 2014-20 plan, while both parties could get at least half the seats in the proposed plan.

5.5 Number of Competitive Districts

An important factor that affects the overall responsiveness of a plan is the number of competitive districts in a plan. I use a variety of approaches to estimate the number of competitive districts in both the 2014-20 State House plan and the proposed plan (see Table 5). Overall, my analysis indicates that the previous plan and the proposed plan are very similar in terms of the number of competitive seats. Moreover, both plans do about as well as the average percentage of seats that are competitive across other states’

elections for their lower chambers in 2020.²⁵

Data:	2020 State House Results	Composite (2014-20)	PlanScore			Mean
Metric:	45-55	45-55	45-55	20%+ Prob. of Each Party Win.	50%+ Prob. Flip in Dec.	
Plan	(1)	(2)	(3)	(4)	(5)	(6)
Average Nationwide in 2020	13%					
2014-20 Plan	13%	24%	23%	20%	25%	21%
Proposed Plan	12%	21%	23%	18%	23%	19%

Table 5: Number of competitive districts using various data sources and metrics.

First, I use the actual 2020 State House results to examine the number of competitive districts. In column 1 of Table 5, I begin by tallying the number of districts where each party’s two-party vote share was between 45 and 55%. This approach indicates that 13% of the districts on the 2014-20 plan were competitive and 12% of the districts on the proposed plan were competitive. It is important to note, however, that a sharp threshold at 55% may not be the best measure of competitiveness.

Next, I use a composite of the 2014-2020 statewide election results to estimate the number of competitive districts. Once again, in column 2 of Table 5, I tally the number of districts where each party’s two-party vote share was between 45 and 55%. This approach indicates that 24% of the districts on the 2014-20 plan were competitive and 21% of the districts on the proposed plan were competitive.

Lastly, I use PlanScore to estimate the potential competitiveness of individual districts on the proposed plan. In column 3 of Table 5, I show the number of districts where PlanScore estimates that each party’s two-party vote share is expected to be between 45 and 55%. This approach indicates that 23% of the districts on the 2014-20 plan were competitive and 23% of the districts on the proposed plan were competitive.

It is also possible to use PlanScore to evaluate whether a district is likely to switch parties at least once per decade (Henderson, Hamel, and Goldzimer 2018). PlanScore conducts 1,000 simulations of possible electoral scenarios based on the results of the 2014-2020 congressional and state legislative elections in every state. Using these simulations,

25. The nonpartisan Princeton Gerrymandering Project gives the proposed plan a low grade on competitiveness. However, their analysis has two material flaws as applied to the proposed plan in Pennsylvania. First, it only uses three recent statewide elections to evaluate competitiveness, and Democrats did unusually well in two of those three elections (2018 Senate and 2018 Governor). Overall, Democrats won 55.3% of the two-party vote in those three elections. Second, it uses a single, very narrow vote share range to classify districts as competitive (46.5-53.5%). Combined, these two assumptions mean that the vast majority of the districts that the Princeton Gerrymandering Project classifies as competitive are unlikely to actually be competitive in a close statewide election. Indeed, Republicans would win the vast majority of these districts. Thus, I do not view the Princeton Gerrymandering Project’s analysis of the plan’s level of competitiveness as a reliable measure of the proposed Pennsylvania State House plan.

PlanScore provides an estimate of the probability that each party will win each seat as well as whether they are likely to have at least a 50% chance of winning each seat once over the course of the decade. In column 4 of Table 5, I estimate the number of districts where each party has at least a 20% chance of winning according to PlanScore. This approach indicates that 20% of the districts on the 2014-20 plan were competitive and 18% of the districts on the proposed plan were competitive. In column 5 of Table 5, I conduct a similar analysis where I tally the number of districts that each party would have at least a 50% chance of winning at least once over the course of the decade. This approach indicates that 25% of the districts on the 2014-20 plan were competitive and 23% of the districts on the proposed plan were competitive.

Finally, column 6 of Table 5 averages across all of these approaches. It indicates that 21% of the districts on the 2014-20 plan were competitive and 19% of the districts on the proposed plan were competitive. Thus, the previous plan and the proposed plan are very similar in terms of the number of competitive seats. The proposed plan also has roughly the same percentage of seats that are competitive as other states' elections for their lower chambers in 2020.

6 Conclusion

This report has evaluated the partisan fairness of the Legislative Reapportionment Commission's proposed Pennsylvania State House plan. Based on three methods of projecting future elections and four different, generally accepted partisan bias metrics, I find that the plan is fair, with just a small pro-Republican bias. On this plan, the party that wins the majority of the votes is likely to usually win the majority of the seats. Thus, the plan satisfies a key premise of democratic theory. Moreover, I find that the plan is much more fair than the 2014-2020 State House plan, which had a large and durable pro-Republican bias. The plan is also likely to be responsive to shifts in voters' preferences.

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